

KUMAUN UNIVERSITY, NAINITAL
Department of Mathematics

M. Sc. Mathematics
(Effective from 2018-19 Batch)

SEMESTERWISE COURSE STRUCTURE AND DETAILED SYLLABUS AS APPROVED BY BOARD OF STUDIES (MATHEMATICS) AND PROPOSED TO BE IMPLEMENTED FROM 2018 BATCH SUBJECT TO THE APPROVAL OF BOARD OF FACULTY AND ACADEMIC COUNCIL:

1. There shall be four semesters in the two- years M.A./M.Sc. Programme in Mathematics.
2. There will be four papers in each semester and one paper comprising viva-voce, comprehensive test and Seminar in semester 4.
3. Each paper will be of 100 marks. This will include a mid-semester/internal assessment of 25 marks in the form of written tests or practical tests in lab oriented courses. In view of the introduction of lab oriented courses, respective mathematics departments may make necessary changes in the intake of students.
4. Viva-voce, comprehensive test and seminar examination of 100 marks will be in semester 4. The board of examiners will consist of one external and one internal examiner recommended for appointment by the BOS.
5. There shall be 400 marks each for semester 1, 2 and semester 3, while 500 marks for semester 4. Thus, for the entire programme the total of marks shall be 1700.
6. Question Paper Structure: Duration of the semester-end examination will be three hours. Each paper in the examination will be of seventy five marks and will comprise of two sections: A and B. Questions within each section will carry equal marks. Section A will be of 30 marks and will contain 7 questions of 6 marks each. The candidate will have to attempt any five questions in this section. Section B will be of 45 marks and shall contain 3 questions. The candidate will have to answer three questions in section C.

Semester wise Course Structure

First semester	Second semester	Third semester	Fourth semester
5311: Real Analysis	5321: Complex Analysis	5331: Linear Algebra	5341: Dynamics of Rigid Bodies
5312 :Topology	5322: Abstract Algebra	5332: Measure Theory	5342: Functional Analysis
5313: Differential Geometry and Tensor Calculus	5323: Differential Equations	5333: Numerical Solutions of ODE and PDE	5343: Calculus of variation and Integral Equations
<i>Elective</i>	<i>Elective</i>	<i>Elective</i>	<i>Elective</i>
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Elective Courses for Odd (First and Third) Semesters:

5351: Mathematical Statistics
5352: Number Theory
5353: Fluid Mechanics
5354: Discrete Mathematics
5355: Computer Programming and Mathematical Computations
5356: Special Functions

Elective Courses for Even (Second and Fourth) Semesters:

5371: Relativity
5372: Riemannian Geometry
5373: Advanced Abstract Algebra
5374: Operations Research
5375: Statistical Analysis
5376: Dynamical Systems

**Abbreviations: C- Compulsory, E- Elective, 4- M. Sc.(Prev.), 5- M. Sc. (Final).
Even and odd numbers represent courses for even and odd semesters respectively.**

DETAILED SYLLABUS for COMPULSORY COURSES

M.A. /M.Sc. (Semester I)

5311: Real Analysis

Unit 1. Metric spaces with various examples, Open sets, Interior of a set, Structure of open subsets of the real line, Limit points, Closed sets, closure of a set, Subspaces.

Unit 2. Cauchy sequences, Complete metric spaces and completion of a metric space, Continuity and Uniform continuity, Sequential notion of continuity and Uniform limit theorem, Compactness.

Unit 3. Functions of several variables: Concept of functions of two variables, Simultaneous and iterated limits in functions of two variables.

Unit 4. Partial derivatives: Definition, Existence and continuity, Interchange of order of differentiation, Directional derivatives.

Unit 5. Composite functions, Continuity and differentiability of functions of two variables, Taylor's Theorem.

Books recommended:

1. S. C. Malik and Savita Arora: *Mathematical Analysis, New Age International.*
2. G.F. Simmons: *Introduction to Topology and Modern Analysis, Tata McGraw Hill.*
3. W. Rudin: *Principles of Mathematical Analysis (3rd edition), Tata Mc Graw Hill Kgakusha, International Student Edition, 1976.*
4. T. M. Apostol: *Mathematical Analysis, Narosa Publishing House, New Delhi, 1985.*

5312: Topology

Unit 1. Topological spaces with examples, Topologies on the real number system, Open sets, Neighbourhood of a point/set.

Unit 2. Local Base, Base and sub-base of a topology, closed sets, interior, boundary, closure, limit point, Derived sets.

Unit 3. Continuous functions, Homeomorphisms, Topological property and topological embedding, Rules for constructing continuous functions in topological spaces.

Unit 4. Compact spaces, Limit point compact and Sequentially compact spaces, Locally compact spaces, Connected spaces, Path connected spaces, Components, Locally connected spaces.

Unit 5. First and Second Countable spaces, Separable and Lindelof spaces, Separation axioms: T_1 , T_2 , T_3 (Regular), T_4 (Normal) spaces.

Books Recommended:

1. J. R. Munkres: *Topology: Narosa Publishing House.*
2. Shaum's outlines series: *Tata McGraw Hill.*
3. K. D. Joshi: *Introduction to General Topology, Wiley Eastern, 1983.*
4. G. F. Simmons: *Introduction to Topology and Modern Analysis, McGraw Hill, 1963.*
5. M. D. Raisinghania & R. S. Aggarwal: *Topology, S. Chand & Co.*

5313: Differential Geometry And Tensor Calculus

Unit 1. Curves in space, parameterized curves, regular curves, helices, arc length, reparametrization (by arc length), tangent, principal normal, binormal, osculating plane, normal plane, rectifying plane, curvature and torsion of smooth curves, Frenet-Serret formulae, Frenet approximation of a space curve.

Unit 2. Order of contact, Osculating circle, osculating sphere, spherical indicatrices, involutes and evolutes, Bertrand Curves, intrinsic equations of space curves, isometries of R^3 , fundamental theorem of space curves, surfaces in R^3 , regular surfaces, co-ordinate neighbourhoods, parameterized surfaces, change of parameters, level sets of smooth functions on R^3 , surfaces of revolution, tangent vectors, tangent plane. first and second fundamental forms, classification of points on a surface.

Unit 3. Curvature of curves on surfaces, normal curvature, principal curvatures, geometric interpretation of principal curvatures, Euler theorem, mean curvature, lines of curvature, Rodrigue's formula, umbilical points, minimal surfaces, definition and examples, Gaussian curvature, intrinsic formulae for the Gaussian curvature, isometries of surfaces,.

Unit 4. Christoffel symbols, curvature tensor, geodesics, geodesics on a surface of revolution, geodesic curvature of a curve,.

Unit 5. n -dimensional real vector space, contravariant vectors, dual vector space, Covariant vectors, tensor product, second order tensors, tensors of type (r, s) , symmetry and skew symmetry of tensors, fundamental algebraic operations: Addition, multiplication, contraction and inner product. Quotient law of tensors.

Books Recommended:

1. C.E. Weatherburn: *Riemannian Geometry and Tensor Calculus.*
2. Andrew Pressley: *Elementary Differential Geometry, Springer (Undergraduate Mathematics Series), 2001.*
3. J. A. Thorpe: *Elementary Topics in Differential Geometry, Springer (Undergraduate Texts in Mathematics), 1979.*
4. D. Somasundaram: *Differential Geometry, A First Course, Narosa Publishing House, New Delhi, 2005.*
5. T.J. Willmore: *An Introduction To Differential Geometry, Oxford University Press.*
6. R.S. Mishra: *A Course in tensors with applications to Riemannian Geometry, Pothishala Pvt. Ltd. Allahabad, 1965.*

M.A. /M.Sc. (Semester II)

5321: Complex Analysis

Unit 1. Conformal mappings, Power series representation of analytic functions, Analytic functions as mappings, Riemann sphere, Linear transformations, Mobius transformation, Cross ratios, Mobius transformation on circles.

Unit 2. Derivative of an analytic function, Higher order derivatives, Cauchy's theorem integral formula. Morera's theorem, Cauchy inequality and Liouville's theorem.

Unit 3. Counting zeros, The open mapping theorem, Maximum modulus principle, Schwarz lemma, The fundamental theorem of algebra.

Unit 4. Harmonic functions, Mean value property, Poisson formula.

Unit 5. Entire functions, Jensen's formula, Meromorphic functions.

Books Recommended:

1. *L.V. Ahlfors: Complex Analysis, Tata McGraw Hill.*
2. *J.B. Conway: Functions of one Complex variable, Springer-Verlag, 1980.*
3. *D. Sarason: Complex Function Theory, Hindustan Book Agency, Delhi, 1994.*
4. *B. Choudhary: Elements of Complex Analysis, Wiley Eastern Ltd., New Delhi, 1993.*

5322: Abstract Algebra

Unit 1. Normal and subnormal series, Zassenhaus's lemma, Schreiers' refinement theorem, composition series, Jordan Holder theorem, chain conditions, examples. Internal and external direct products and their relationship.

Unit 2. Sylow subgroups. Sylow's I, II and III theorems, p – groups, examples and applications, Groups of order p q , Direct and inverse images of Sylow subgroups.

Unit 3. Commutators. Solvable groups, solvability of subgroups and factor groups. Nilpotent groups and their equivalent characterisations.

Unit 4. Rings, ideals, prime and maximal ideals, quotient rings. Factorisation theory in commutative domains. Prime and irreducible elements, G.C.D. Euclidean Domains. Principal Ideal Domain. Divisor chain condition. Unique Factorisation Domains, examples and counter examples. Polynomial rings over domains. Eisenstein's irreducibility criterion. Unique factorisation in polynomial rings over U.F.D.s.

Unit 5. Fields, finite fields, field extensions, Galois extensions.

Books Recommended:

1. *J. Gallian: Abstract Algebra, Narosa Publication.*
2. *N. Jacobson: Basic Algebra, Vol. I, Hindustan Publishing Co., New Delhi.*
3. *M. Artin: Algebra, Prentice Hall of India.*
4. *Ramji Lal: Fundamentals in Abstract Algebra, Chakra Prakashan, Allahabad, 1985.*
5. *I. N. Herstein: Topics in Algebra, Wiley Eastern Ltd., N.D., 1975.*
6. *D. S. Dummit and R. M. Foote: Abstract Algebra, John Wiley, N. Y.*

5323: DIFFERENTIAL EQUATIONS

Unit 1. Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs., Sturm-Liouville boundary value problem, Green's function.

Unit 2. Formation of P.D.E.'s. First order P.D.E.'s, Classification of first order, P.D.E.'s, Complete, general and singular integrals, Lagrange's or quasi-linear equations, Integral surfaces through a given curve. Orthogonal surfaces to a given system of surfaces, Characteristic curves.

Unit 3. Charpit's method, Jacobi's Method. Cauchy problem for first order PDEs.

Unit 4. Linear equations with constant coefficients, Reduction to canonical forms, Classification of second order P.D.E.s. General solution of higher order PDEs with constant coefficients.

Unit 5. Method of separation of variables: Laplace, Heat and Wave equations in Cartesian, cylindrical and spherical polar coordinates.

Books Recommended:

1. *M. D. Raisinghania, Advanced Differential Equations, S. Chand, 2016.*
2. *D.P. Choudhary and H. I. Freedman: A Course in Ordinary Differential Equations, Narosa Publishing House, New Delhi, 2002.*
3. *E.A. Coddington: AN Introduction to Ordinary Differential Equations, Prentice Hall of India, New Delhi, 1968.*
4. *T. Amaranath: An Elementary Course in Partial Differential Equations, Narosa Publishing House, New Delhi, 2005.*
5. *Erwin Kreyszig: Advanced Engineering Mathematics, John Wiley &SON Inc., New York, 1999.*

M.A. /M.Sc. (Semester III)

5331: Linear Algebra

Unit 1. A brief review of vector space, Inner products, Orthogonality, Best approximations, Projections, Cauchy-Schwartz inequality.

Unit 2. Adjoint of a linear transformation, Self adjoint transformations, Unitary operators.

Unit 3. Normal operators: Definition and properties.

Unit 4. Spectral theory for normal operator, Polar decomposition of a linear operator, Roots of a family of normal operators, Self adjoint algebra generated by a family of linear operators.

Unit 5. Eigen vectors and eigen values of a linear operator, Minimal polynomial of a linear operator and its relations to characteristic polynomial, Caley-Hamilton theorem.

Books Recommended:

1. Hadley: *Linear Algebra*.
2. Hoffman and Kunz: *Linear Algebra, Prentice Hall of India, New Delhi, 1972*.
3. H. Helson: *Linear Algebra, Hindustan Book Agency, New Delhi, 1994*.
4. K. B. Dutta: *Matrix and Linear Algebra, Prentice Hall of India*.

5332: Measure Theory and Integration

Unit 1. Countable sets, uncountable sets, relation between the cardinality of a nonempty set and the cardinality of its power set; Boolean ring, σ -ring, Boolean algebra and σ -algebra of sets, Set function.

Unit 2. Lebesgue Measure: Introduction, Outer measure, Measurable sets and Lebesgue measure, Example of nonmeasurable sets, Measurable functions.

Unit 3. The Lebesgue Integral: The Riemann integral, The Lebesgue integral of a bounded function over a set of finite measure, The integral of nonnegative functions. The general Lebesgue integral, Convergence in measure.

Unit 4. Differentiation and Integration: Differentiation of monotone functions, Functions of bounded variation, Differentiation of an integral, Absolute continuity, Convex functions.

Unit 5. General Measure and Integration Theory: Measure spaces, Measurable functions, Integration, General convergence theorems, The L^p spaces, Measure and Outer Measure, Outer measure and measurability, The extension theorem, Inner measure, Caratheodory outer measure.

Books Recommended:

1. P. K. Jain: *Measure Theory, New Age International*.
2. P. R. Halmons: *Measure Theory, Grand Text Mathematics, 14 Springer, 1994*.
3. E. Hewit and K. Stromberg: *Real and Abstract Analysis, Springer, 1975*.
4. K.R. Parthasarathy: *Introduction to Probability and Measure, TRIM 33, Hindustan Book Agency, New Delhi, 2005*.
5. I. K. Rana: *An Introduction to Measure and Integration, (Second Edition), Narosa Publishing House, New Delhi, 2005*.

5333: Numerical Solutions of ODE and PDE

Unit 1. Numerical Solution of ordinary Differential equations: Numerical solution of ODE by Picard's, Euler's, Modified Euler's and Runge-Kutta methods, Boundary value problems: Finite difference method, Shooting method.

Unit 2. Numerical Solution of Partial Differential equations: Classification of second order general PDE, Difference method.

Unit 3. Difference methods for Parabolic PDE. Heat conduction equation and its numerical solutions with finite difference methods (Two and three level difference methods).

Unit 4. Difference methods for Hyperbolic PDE. Wave equation and its numerical solutions with finite difference methods (First order only).

Unit 5. Difference methods for Elliptical PDE. Dirichlet problem for Laplace equation and its numerical solutions with finite difference methods.

Practical assignments: Based on topics included in the paper with Matlab/OCTAVE, Calculator is allowed in the Examination.

Books Recommended:

1. S. S. Sastry: *Introductory Methods Numerical Analysis, Prentice- Hall of India*.
2. M. K. Jain, S R K Iyengar, R K Jain: *Computational Methods for Partial Differential equations: New Age international, New Delhi, 2016*.
3. M. K. Jain: *Numerical Solutions of Differential Equations: New Age international, New Delhi, 2014*.
4. C. F. Gerald and P. O. Wheatley: *Applied Numerical Analysis, Addison- Wesley, 1998*.

M.A. /M.Sc. (Semester IV)

5341: Dynamics of Rigid Bodies

Unit 1. D'Alembert's principle, Motion about a fixed axis (Finite and Impulsive forces).

Unit 2. Motion in two dimensions under Finite and Impulsive forces.

Unit 3. Principle of conservation of momentum and energy.

Unit 4. Lagrange's equations in generalized co-ordinates.

Unit 5. Hamilton's principle, principle of least action, Euler's geometrical and dynamical equations.

Books Recommended:

1. S. L. Loney: *Dynamics of rigid bodies*.
2. Bhu Dev Sharma: *Dynamics of Rigid Bodies*, Kedarnath Ramnath Sons, 1984.
3. M. Ray & Harswarup Sharma: *A text book of Dynamics of Rigid Body*, Students' Friends & Co., Agra-2, 1971.
4. A. S. Ramsey: *Dynamics – Part II*.
5. H. Goldstein: *Classical Mechanics*, Narosa, 1990.

5342: Functional Analysis

Unit 1. Inequalities (Auxillary, Cauchy Schwarz, Holder and Minkowski), Examples of metric spaces (especially R^n , C^n , l^∞ , $C[a, b]$, s , $B(A)$, l^p).

Unit 2. Normed and Banach Spaces, Completion of a normed space, Finite dimensional normed spaces; Compactness and finite dimension, linear operators, Bounded and continuous linear operators; Linear functional; linear operators and functional on finite dimensional spaces, Dual space.

Unit 3. Inner product space; Hilbert space; Properties of Inner product spaces, Orthogonal complements and direct sums, Orthonormal sets and sequences; Hilbert adjoint operators, Self-Adjoint, Unitary and normal operators.

Unit 4. Zorn's Lemma, Hahn Banach Theorem for real vector, Open mapping theorem, Closed graph theorem.

Unit 5. Banach Contraction Principle (BCP), Some applications of BCP.

Books Recommended:

1. Erwin Kreyszig: *Introductory Functional Analysis*, Wiley India edition.
2. G. F. Simmons: *Introduction to Topology and Modern Analysis*, McGraw Hill, 1963.
3. E. M. Stein & R. Shakarchi: *Functional Analysis*, Princeton University Press, 2000.
4. B. V. Limaye: *Functional Analysis*, New Age International Publisher, 2014.

5343: Calculus of Variations and Integral equations

Unit 1. Functionals and extremals, Necessary and sufficient conditions for extrema, Variation and its properties.

Unit 2. Euler equations, Cases of several dependent and independent variables, Variational methods for boundary value problems in ordinary and partial differential equations, Functionals dependent on higher derivatives, Parametric forms, Simple applications.

Unit 3. Classification of linear integral equations, Relation between differential and integral equations.

Unit 4. Fredholm equations of second kind with separable kernels, Fredholm alternative theorem, Eigen values and eigen functions.

Unit 5. Method of successive approximation for Fredholm and Volterra equations, Resolvent kernel.

Books Recommended:

1. L. Elsgolts: *Differential Equations and Calculus of Variations*, Mir Publishers, 1970.
2. A. S. Gupta: *Calculus of Variations*, Prentice Hall of India, New Delhi, 1999.
3. J. H. Davis: *Methods of Applied Mathematics with a MATLAB Overview*, Birkhäuser, Inc., Boston, MA, 2004.
4. L.G. Chambers: *Integral Equations A short Course*, Int. Text Book company Ltd. 1976.
5. Abdul J Jerry: *Introduction to Integral Equations with Applications*, Marshal and Dekkar.
6. Naveen Kumar: *An Elementary Course on Variational Problems in Calculus*, Narosa, 2004.

5344: Viva-Voce, Comprehensive Test and Seminar

1. In this paper evaluation will be based on the student's performance in viva voce, comprehensive test and presentation /seminars on any current topic in mathematics.
2. The respective departments will decide the schedule of conducting the seminars during the fourth semester and a hardcopy of the same will be submitted by the students at the time of viva voce examination.
3. Marks for the presentation/seminars will be allotted out of Twenty five marks.
4. The Viva-Voce examination and comprehensive test will be conducted together.
5. The subject comprehensive test will be of 25 marks consisting of short answer questions.
6. During the viva-voce examination subject knowledge of the students based on the courses studied during the course program will be tested by the examiners. It will be graded out of fifty marks.
7. There shall be one external and one internal examiner to conduct the Viva-voce examination.

Elective Courses for Odd (First and Third) Semesters

5351: Mathematical Statistics

Unit 1. Descriptive Statistics: Measures of central tendency, dispersion skewness and kurtosis Elements of probability: Sample space, discrete probability, independent events, Baye's theorem, random variables and distribution functions (univariate, bivariate, and generalization to multivariate).

Unit 2. Mathematical expectation and moments: Moment generating function, Characteristic function and cumulants. Probabilistic inequalities (Tchebychev, Markov and Jensen). Modes of convergence: weak and strong laws of large numbers. Central limit theorem (i.i.d. case). Markov chains with finite and countable state space, Poisson and birth- and- death processes.

Unit 3. Some standard discrete and continuous univariate distributions (Binomial, Poisson, Normal, Gamma and Beta).

Unit 4. Correlation, Rank correlation. Regression lines. Multiple and partial correlation of three variables only.

Unit 5. Concept of sampling and statistics: simple random sampling, Stratified sampling and systematic sampling, Probability proportional to size sampling, Ratio and regression methods.

Books Recommended:

1. *M.G.Kendall: Advanced theory of statistics Vol. I &II, Charle's Griffiin & Co.*
2. *R. Hogg and A Craig: Introduction to Mathematical Statistics, Mac Millan & Co.*
3. *W.C. Cochran: Sampling techniques, Wiley Eastern, Reprint.*
4. *C.E. Weatherbun: A first course in Mathematical Statistics, The English Language Book Society And Cambridge University Press, 1961.*
5. *John A. Rice: Mathematical Statistics and Data Analysis, (3rd Edition), Durbury, 2013.*
6. *C.R. Rao: Advanced statistical methods in Biometrical Research, John Wiley.*
7. *S.C. Gupta & V.K. Kapoor: Fundamentals of Mathematical Statistics, Sultan Chand & Co.*

5352: Theory of Numbers

Unit 1. Divisibility theory in integers, Prime Numbers, Unique Factorization theorem.

Unit 2. Theory of congruences, Fermet's theorem, Wilson's theorem.

Unit 3. Number-theoretic functions: $d(n)$, $\sigma(n)$, $\mu(n)$ and $\varphi(n)$ including elementary results.

Unit 4. Primitive roots, Residues, Quadratic Reciprocity Law, Perfect numbers.

Unit 5. Fibonacci numbers, Continued fractions, Irrational numbers, Representation of numbers by two or four squares.

Books Recommended:

1. *G. H. Hardy and E. M. Wright: Introduction to the theory of numbers, Oxford University Press, 4th Edition.*
2. *D. M. Burton: Elementary Number Theory, 6th Edition, Tata McGraw Hill.*
3. *Thomas Koshy: Elementary Number Theory with Applications, Academic Press, 2nd Edition.*
4. *Kenneth H. Rosen: Elementary Number Theory and its Applications, Addison-Wesley Publishing Company, 1986.*

5353: Fluid Dynamics

Unit 1. Lagrangian and Eulerian methods, Equation of continuity, Boundary surface, Stream lines, Velocity potential, Euler's equation of motions, Bernoulli's theorem, Helmholtz equations, Cauchy's integral, Equation of motion under impulsive forces, Principal of energy.

Unit 2. Motion in two dimensions, Velocity potential and current functions, Sources and sinks, Doublet and images, Circle theorem, Motion of circular and elliptic cylinder in two dimensions, Joukowski transformation, Motion in three dimensions, Three dimensional sources, Sinks and doublets, Image of source in front of sphere, Motion of spheres, Stroke's stream function.

Unit 3. General theory of irrotational motion, Permanence of irrotational motion circulation, Stroke's theorem, Kelvin's circulation theorem, Green's theorem, Kelvin's minimum energy theorem, Conformal Representation, Kutta and Joukowski transformation,.

Unit 4. Vortex motion: Rectilinear vortices, Rectilinear vortex with a circular section, An infinite row of parallel rectilinear vortices, Karman stream, Use of conformal transformation, Vortex pairs.

Unit 5. General theory of stress strain, Navier-Stroke's equations.

Books Recommended:

1. *A. S. Ramsey: A Treatise on Hydrodynamics.*
2. *W. H. Besant and A. S. Ramsey: A Treatise on Hydrodynamics, CBS Publisher and Distributors, Delhi, 1988.*
3. *F. Chorlton: A Text Book of Fluid Dynamics, CBC, 1985.*
4. *S. W. Yuan: Foundations of Fluid Dynamics, Prentice-Hall of India, 1988.*
5. *M. D. Raisinghania: Fluid Dynamics, S. Chand, 1939.*

5354: Discrete Mathematics

Unit 1. Principle of mathematical induction, Partially ordered sets, Lattices: Lattices as partially ordered sets, Their Properties, Lattices and algebraic systems. Principle of duality, Sub lattices, Complete, Complemented and Distributive lattices.

Unit 2. Boolean algebra, Boolean functions, Boolean expressions, Applications to switching circuits.

Unit 3. Elements of graph theory: Basic terminology, Paths and circuits, Eulerian and Hamiltonian graphs, Planar graphs, Directed graphs.

Unit 4. Trees: Rooted trees, path lengths, spanning trees, minimum spanning trees.

Unit 5. Permutations and Combinations, the rules of sums and products, Properties of binary relations, Equivalence relations and partitions, Functions and Pigeonhole principle, Principle of inclusion and exclusion.

Books Recommended:

1. C. L. Liu: *Elements of discrete mathematics*, Tata McGraw Hill Education, 2008.
2. Ram Babu: *Discrete Mathematics*, Pearson Edition India, 2011.
3. Lipschutz: *Discrete Mathematics*, Tata McGraw Hill, 2011.

5355: Computer Programming and Mathematical Computation

Unit 1. Introduction to Programming in C: Introduction to Algorithms & Flowcharts Variables, constant, Keywords, signed and unsigned modifiers.

Unit 2. Expression and operators: Arithmetic, logical and relational operators, bitwise operators, incremental operators, assignment operators. Functioning of these operators.

Control flow: If-else, switch, while, do-while, for loops, continue, break statements, Nesting of control statements and loops.

Unit 3. Working with functions: Variable and functions, Argument passing to functions, type of functions, storage classes, scope rule, C preprocessor and standard libraries.

Unit 4. Pointers, arrays and File handling: Pointers, addresses, arrays, multidimensional arrays, String, Input/ Output, Standard input and output, basic file handling.

Unit 5. User Defined Data-types : Structure, Union, enumeration.

Practical assignments: Based on topics included in the paper.

Books recommended:

1. Ritchie & Kernighan: *The C programming language*, Prentice Hall of India.
2. V. Rajaraman: *Computer Programming in 'C'*, Prentice Hall of India.
3. E. Balaguruswami: *Programming in ANSI 'C'*, Tata McGraw Hill.

5356: Special Functions

Unit 1. Preliminaries, Gamma function and related functions, Gauss multiplication theorem, the hypergeometric differential equation, Gauss hypergeometric function.

Unit 2. Integral representation of hypergeometric function, Evaluation of hypergeometric function, the confluent hypergeometric differential equation, Confluent hypergeometric function.

Unit 3. Bessel's equation, solution of Bessel's equation, Bessel's functions $J_n(x)$, Recurrence Formulae, Equations reducible to Bessel's equation, orthogonality of Bessel's Functions, A generating function for $J_n(x)$, Basic properties.

Unit 4. Legendre's equation, Legendre's polynomial $P_n(x)$, Legendre's function of the second kind $Q_n(x)$, General solution of Legendre's equation, Rodrigue's formula.

Unit 5. Legendre polynomials, A generating function of Legendre's polynomial, Orthogonality of Legendre polynomials, Recurrence formulae for $P_n(x)$.

Books Recommended:

1. E.D. Rainville: *Special functions*.
2. Nirvikar Saran: *Special Functions*.
3. W.W. Bell: *Special Function for Scientists and Engineers*, Dever publications, 2002,
4. U.P. Singh: *Special Function & Their Application*, WISDOM PRESS, 2012.

Elective Courses for Even (Second and Fourth) Semesters

5371: Relativity

Unit 1. Special Relativity: Inertial Frames of reference, Michelson-Morley experiment, Doppler effect, Stellar aberration, Simultaneity, Postulates of special relativity, Lorentz transformation, Length contraction, Time dilation, Clock paradox, Addition of velocities and accelerations, Four-dimensional space time, Light cone, Mass variation, Velocity four vector, Momentum and force, Mass-Energy relationship.

Unit 2. General Relativity: Geodesics, Geodesic coordinates, Curvature tensor and its algebraic properties, Bianchi's identities, Contracted curvature tensor, Conditions for a flat space time, Displacement of space-time, Killing equations, Groups of motion, Space-time of constant curvature.

Unit 3. Principal of covariance, Non-inertial frames of reference, Principal of equivalence, Weak field approximation of geodesic equations, Law of gravitation in empty space-time, Canonical coordinates, Schwarzschild solutions.

Unit 4. Experimental tests of general relativity, Schwarzschild metric in isotropic coordinates, Birkhoff's theorem, Law of gravitation in non-empty space time.

Unit 5. Energy-Momentum tensor for a perfect fluid, Poisson's equation as the weak field approximation, Schwarzschild interior solution, Gravitational collapse of a ball. Einstein-Maxwell equations of electromagnetism, Gravitational field of a point charge.

Books Recommended:

1. D.F. Lawden: *An Introduction to tensor calculus and relativity*,
2. J.V. Narlikar: *General relativity and cosmology*.
3. R.H. Good: *Basic concept of relativity*, 1978.
4. A.S. Eddington: *Mathematical theory of relativity*, 1981.

5372: Riemannian Geometry

Unit 1. Dual vector Spaces : N-dimensional real vector space, Covariant vectors, Dual space, Contravariant vectors, tensor product, Other tensors of second order, Tensors of type (r,s). Algebraic Operations on tensors: Symmetric and skew symmetric properties, Fundamental algebraic operations, Inner product of vectors, Euclidean vector space.

Unit 2. Tensor Calculus : Differentiable manifold, Lie-bracket, Tangent space, Connexions, Covariant derivatives, Curvature tensor, Parallelism. Lie derivative, Exterior derivative, Cartan's structural equations.

Unit 3. Riemannian geometry : Riemannian metric, Christoffel symbols, Curvature tensor with respect to Christoffel symbols, Differential operators, Geodesics, Geodesic coordinates, Riemannian curvature, Conformal curvature tensor, Frenet's formulae.

Unit 4. Ricci's Coefficients of Rotation : Orthonormal basis, Curl of a congruence, Canonical congruences, Gaussian and Ricci curvature.

Unit 5. Sub-manifolds and Hypersurfaces : Normals, Gauss's formulae, Weingarten equations, Coordinate viewpoint, Lines of curvature, Generalized Gauss and Mainardi-Codazzi equations.

Books Recommended:

1. R.S. Mishra: *A Course in tensors with applications to Riemannian Geometry*, Pothishala Pvt. Ltd., Allahabad, 1965.
2. K. Yano: *The theory of Lie derivatives and its applications*, North-Holland Publishing Company, Amsterdam, 1957.
3. Matthew S. Smith: *Principal and Application of Tensor Analysis*, W. Sons (Indianapolis) 1963.
4. N. J. Hicks: *Notes on differential geometry*, Van Nostrand publishing.
5. H.S. Shukla, Prasad & Dhruwa Narain Dubey: *Differential Geometry of Manifolds*, Vandana Prakashan, Mohanlalpur, Gorakhpur.

5373: Advanced Abstract Algebra

Unit 1. Modules over a ring. Endomorphism ring of an abelian group. R – Module structure on an abelian group M as a ring homomorphism from R to $\text{End}_Z \{M\}$. Submodules. Direct summands. Annihilators. Faithful modules. Homomorphism. Factor modules. Correspondence theorem, Isomorphism theorems.

Unit 2. $\text{Hom}_R[M, N]$ as an abelian group and $\text{Hom}_R[M, M]$ as a ring. Exact sequences. Five lemma. Products, coproducts and their universal property. External and internal direct sums.

Unit 3. Free modules. Homomorphism extension property. Equivalent characterisation as a direct sum of copies of the underlying ring. Split exact sequences and their characterisations. Projective modules. Injective modules. Divisible groups. Examples of injective modules. Boolean Algebra.

Unit 4. Factorisation of polynomials in extension fields. Splitting fields and their uniqueness. Seperable field extensions. Perfect fields. Seperability over fields of prime characteristic. Transitivity and seperability. Automorphism of fields. Dedekind's theorem. Fixed fields. Normal extensions. Splitting fields and normality. Normal closures.

Unit 5. Galois extensions. Fundamental theorem of Galois theory. Computation of Galois groups of polynomials.

Books Recommended:

1. J. A. Gallian, *Contemporary Abstract Algebra*, Narosa Publication, 7th Edition.
2. Vivek Sahai, and Vikas Bisht: *Algebra*, Narosa Publishing House 1999.
3. I. N. Herstein: *Topics in Algebra*, Wiley Eastern, 1963.

5374: Operations Research

Unit 1. Integer Programming: Pure and Mixed integer programming, Gomory all IPP method, Fractional cut method, Branch and bound method.

Unit 2. Sensitivity Analysis: Changes in Objective Function Coefficient, Changes in constants, Changes in coefficients of decision variables in constraints, Structural changes.

Unit 3. Network Analysis: Network flow problem, minimal spanning tree problem, shortest rout problem, maximal flow problem, minimum cost flow problems, critical path analysis, PERT and CPM

Unit 4. Nonlinear Programming, Formulation of NLPP, general NLPP, constrained optimization with equality and inequality constraints.

Unit 5. Game Theory: Two persons Zero sum Games, Max-min/ Min-Max Principle, Games without saddle points, Graphical solutions, Dominance property.

Books Recommended:

1. H. A. Taha: *Operations Research, An Introduction*, Pearson.
2. Kanti Swarup, P K Gupta, Manmohan: *Operations Research, Sultan Chand & Sons, New Delhi*.
3. S.S. Rao: *Optimization Theory and Applications Wiley Eastern*.
4. F. S. Hiller and G. J. Leiberman: *Introduction to Operation Research (6th Edition), McGraw-Hill International Edition, 1995*.

5375: Statistical Analysis

Unit 1. Statistical Inference: Concept of consistency, efficiency, sufficiency, unbiasedness, and completeness. Existence of best asymptotically, normal estimates under regulatory conditions.

Unit 2. Maximum likelihood and other methods of estimation. Properties of maximum likelihood estimates. Minimax and Baye's estimates. Interval estimation: Neyman's Approach. Best confidence intervals.

Unit 3. Testing of Hypothesis: Simple and composite hypothesis, critical region, two types of errors, level of significance and power of a test. Most powerful test and uniformly most powerful test.

Unit 4. Neyman and Pearson's lemma. Likelihood Ratio tests. Large sample test. Sampling distribution of mean and variates. Exact sampling distributions: t, F and Z distributions and tests of significance based on them. Chi square distribution and its applications.

Unit 5. Non parametric tests. Analysis of variance and covariance. Gauss – Markov models. Fixed, random and mixed effect models.

Books Recommended:

1. J. Medhi: *Stochastic Processes, Wiley Eastern Ltd.*
2. H.C. Saxena & P.U. Surendran: *Statistical Inference, S. Chand & Co.*
3. H.C. Saxena: *Mathematical Statistics .*
4. S.C. Gupta & V.K. Kapoor: *Fundamentals of Mathematical Statistics, Sultan Chand & Co.*
5. S.K. Sinha: *Reliability & Life Testing.*

5376: Dynamical Systems

Unit 1. One Dimensional Dynamics: Examples of dynamical systems, Preliminaries from calculus, elementary definitions, Hyperbolicity, An example from quadratic family, symbolic dynamics.

Unit 2. Topological conjugacy, Chaos, structural stability, Sarkovskii's theorem, The Schwarzian derivative, Bifurcation theory.

Unit 3. Complex Analytic Dynamics: Preliminaries from complex analysis, The Riemann sphere, Steriographic projection, Examples from quadratic maps.

Unit 4. Equicontinuity and normal families, Montel's Theorem, Julia and Fatou sets, Fixed and periodic points and their classification.

Unit 5. Critical points, Exceptional points, Properties of Julia sets, Mandelbrot set.

Books Recommended:

1. R. L. Devaney: *An Introduction to Chaotic Dynamical Systems, Addison- Wesley.*
2. A. F. Beardon: *Iteration of Rational Functions, Springer- Verlag.*
3. C. G. Carlson and T. W. Gamelin: *Complex Dynamics, Universitext, Springer.*
4. R. A. Holmgren: *A first course in discrete dynamical systems, Springer.*